NDRC
A-41M

Send to J Feldman 762

[redacted]

NATIONAL DEFENSE RESEARCH COMMITTEE

MEMORANDUM NO. A-41M

A NOTE ON VON KÁRMÁN'S THEORY OF THE PROPAGATION OF PLASTIC DEFORMATION IN SOLIDS

OSRD #664

by

H.F. Bohnenblust

[redacted]

[redacted]

Copy No. 31

NDRC
A-41M

UNCLASSIFIED

[redacted]

NATIONAL DEFENSE RESEARCH COMMITTEE

MEMORANDUM NO. A-41M

A NOTE ON VON KÁRMÁN'S THEORY OF THE PROPAGATION OF PLASTIC DEFORMATION IN SOLIDS

by

H.F. Bohnenblust

DISTRIBUTION STATEMENT F:
Further dissemination only as directed by
OSRD
Washington DC 20330
or higher DoD authority.

JUN 1942

Approved on June 17, 1942
for submission to the Section Chairman

H P Robertson
H.P. Robertson, Vice Chairman
Section B, Division A

Approved on June 24, 1942
for submission to the Division Chairman

L E Burchard
L.E. Burchard, Chairman
Section B, Division A

Approved on June 25, 1942
for submission to the Committee

Richard C. Tolman
Richard C. Tolman, Chairman
Division A

[redacted]

UNCLASSIFIED

The work described in this memorandum is pertinent to the projects designated by the War Department Liaison Officer as CE-5 and CE-6, to the project designated by the Navy Department Liaison Officer as NO-11, and to Division A project PA-115.

The work herein described was carried out at Princeton University under Contract OEMsr-260. The original manuscript for this memorandum was transmitted to Division A on June 9, 1942. The memorandum in its final form incorporates minor alterations which were approved by the author on June 17.

Distribution of copies of this memorandum. -- The editorial staff of Division A completed preparation of the memorandum for duplication on June 25, 1942. The initial distribution of copies was as follows:

Copies No. 1 to 24, inclusive, to the Office of the Secretary of the Committee for distribution in the usual manner;

Copy No. 25 to the Chief of the Bureau of Ordnance, Navy Department (Attention: Research and Development Division);

Copy No. 26 to the Corps of Engineers (Attention: Fortifications Section, Lt. Col. F.J. Wilson);

Copy No. 27 to the Bureau of Yards and Docks (Attention: Planning and Design Department, Capt. C.A. Trexel);

Copy No. 28 to the David Taylor Model Basin (Attention: Comdr. W.P. Roop);

Copy No. 29 to Frankford Arsenal (Attention: Lt. Col. L.S. Fletcher);

Copy No. 30 to Watertown Arsenal (Attention: Col. H.H. Zornig);

Copy No. 31 to Aberdeen Proving Ground (Attention: Ballistics Research Laboratory, Lt. Col. L.E. Simon);

Copy No. 32 to U.S. Naval Proving Ground (Attention: Lt. Comdr. R.A. Sawyer);

Copy No. 33 to U.S. Naval Research Laboratory (Attention: Ross Gunn);

Copy No. 34 to J.E. Burchard, Chairman, Section B, Division A;

Copy No. 35 to H.P. Robertson, Vice Chairman, Section B, Division A;

Copy No. 36 to Th. von Kármán, Consultant, Section B, Division A;

Copy No. 37 F. Seitz, Member, Section B, Division A;

Copy No. 38 to J.B. Wilbur, Member, Section B, Division A;

Copy No. 39 to N.M. Newmark, Consultant, Section B, Division A;

Copy No. 40 to R.J. Slutz, Technical Aide, Section B, Division A.

A NOTE ON VON KÁRMÁN'S THEORY OF THE PROPAGATION OF PLASTIC DEFORMATION IN SOLIDS

Preliminary experiments were carried out by Pol. E. Duwez[1] to check the formulas established by Th. von Kármán in his theory of the propagation of plastic deformation in solids.[2] Theoretical and experimental results are compared in Figs.5, 12 and 13 of Duwez's report, and it is pointed out that in Figs.12 and 13 a discrepancy occurs. It is the purpose of the present memorandum to show that this discrepancy between theory and experiment is explained to a great extent if two changes are made in the interpretation of the theoretical results. These changes consist simply in avoiding two simplifications which were made by von Kármán and Duwez, who underestimated their relative importance.

1. Statement of problem; notation

A rod extending from $x = -\infty$ to $x = 0$, of density ρ_0 and cross section A_0 is at rest and in an unstrained condition. At time $t = 0$, its end $x = 0$ is suddenly put into motion with a constant velocity v_1. The function $u(x,t)$ denotes the displacement at time t of that element of the rod which, at time $t = 0$, had the position x. Thus, during the course of the motion any element is labeled with the same value of x that it had at the beginning of

---

1/ Duwez, Preliminary experiments on the propagation of plastic deformation, NDRC Report A-33 (OSRD No. 380)

2/ Von Kármán, On the propagation of plastic deformation in solids, NDRC Report A-29 (OSRD No. 365).

the motion. Then

$$\varepsilon = \varepsilon(x,t) = \partial u/\partial x$$

is the strain and

$$v = v(x,t) = \partial u/\partial t$$

is the velocity of this element at time $\underline{t}$. Finally, let

$$\sigma = \sigma(\varepsilon)$$

be the stress and put the "velocity" $\underline{c}(\varepsilon)$ equal to

$$c(\varepsilon) = \left[\frac{1}{\rho_0}\frac{d\sigma}{d\varepsilon}\right]^{\frac{1}{2}}.$$

2. Discussion

The first change in the theoretical results consists in interpreting $\underline{\sigma}$ in von Kármán's formulas as the applied stress σ_{app} rather than as the true stress σ_{true} considered by Duwez in his report. Let

$$F = \sigma_{true} \cdot A = \sigma_{app} \cdot A_0$$

be the tensile force in the rod as a function of $\underline{x}$ and $\underline{t}$; here $\underline{A}$ is the area of the actual cross section of the rod while it is under strain. In setting up his differential equation (1), namely,

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{d\sigma}{d\varepsilon}\frac{\partial \varepsilon}{\partial x},$$

von Kármán considered the mass,

$$m = \rho_0 A_0 \Delta x,$$

which at $t = 0$ was situated between x and $x + \Delta x$. This mass is subjected to the resultant force

$$F(x + \Delta x, t) - F(x,t) = (\partial F/\partial x)\Delta x.$$

Thus

$$\rho_0 A_0 \Delta x \frac{\partial^2 u}{\partial t^2} = \frac{\partial F}{\partial x}\Delta x = A_0 \frac{\partial \sigma_{app}}{\partial x}\Delta x = A_0 \Delta x \frac{d\sigma_{app}}{d\varepsilon}\frac{\partial \varepsilon}{\partial x},$$

and we see that σ_{app} is the stress entering in Eq.(1). The importance of the distinction between A and A_o was first realized by Dr. C. Zener and Lt. J.H. Holloman.[3/]

The second change in the interpretation of the theoretical results arises in connection with the shock wave produced by the stopping of the end point x = 0 of the rod at the end of the time of impact. The precise behavior of this shock wave is beyond our present knowledge; hence, for a comparison of theory with experiment, certain simplifying assumptions must be made.

Von Kármán assumed that at time t = T -- that is, at the end of the impact -- the rod is "frozen" with the displacements u(x,T). We assume that the rod is "frozen," not at time T, but at the time of passage of a shock wave traveling with a speed corresponding to the slope −c(0) = −12,200 ft/sec on the x,t-diagram. Thus the rod will show the displacements

$$u\left(x, T - \frac{x}{c(0)}\right). \qquad [x \leq 0]$$

This shock wave meets the plastic wave at the point

$$x = -c_o c_1 T/(c_o - c_1)$$

and at the time

$$t = c_o T/(c_o - c_1).$$

Here $c_o = c(0)$ and c_1 is the value $c(\varepsilon_1)$ for which

$$\int_o^{\varepsilon_1} c(\varepsilon)d\varepsilon = v_1.$$

3/ This work was called to our attention by Prof. Walker Bleakney, who learned of it during a conference held at Watertown Arsenal. The work has since been reported in NDRC Memo. A-37M.

The assumption that the $\underline{x}, \underline{t}$-curve for the shock wave has a slope equal to $-c(0)$ is not fully justified theoretically, but corresponds to the assumption that the unloading curve — the curve of $\underline{\varepsilon}$ versus $\underline{\sigma}$ — is a straight line whose slope is equal to Young's modulus.

LeVan Griffis and M.P. White[4/] have attempted to determine the distribution of strain and velocity for any given boundary condition at $x = 0$. [They prescribe $\varepsilon(x,t)$ for $x = 0$ as a function of $\underline{t}$ rather than prescribe $v(x,t)$ for $x = 0$, as in von Kármán's report; this is not essential, of course.] To obtain this solution they make use of the superposition principle, which is, however, not generally applicable to nonlinear problems. Nevertheless, it is true that the superposition principle leads to the correct solution as long as the prescribed boundary value function $\varepsilon(0,t)$ increases with $\underline{t}$. It cannot be used for the stopping mechanism, or, at least, it is not clear to what extent it can be so used.[5/]

3. Comparison of theory and experiment

The three figures of the present memorandum are numbered to correspond to Figs.12, 13 and 5 of Duwez's report.

4/ In a manuscript made available to the National Defense Research Committee.

5/ White and Griffis, in a revised version of the paper cited in reference 4, have succeeded in computing the permanent strains in the rod. It is expected that their results, together with additional relevant work by the present author, will be published in this series. [Note added in proof, June 15, 1942.]

Figure 12. -- The curves shown in Fig.12 are obtained by plotting the derivative of the experimentally observed displacements as a function of the position $\underline{x}$. The derivative is the total derivative,

$$\frac{d}{dx}\left(x, T - \frac{x}{c_0}\right) = \varepsilon - (v/c_0).$$

Thus our Fig.12, as compared with Fig.12 of Duwez's report, has an additive correction $-v/c_0$ which amounts to more than 10 percent of the value of $\underline{\varepsilon}$.

Figure 13. -- In this figure also the ordinate is $\varepsilon-(v/c_0)$ instead of $\underline{\varepsilon}$. The abscissa is the quotient of the $\underline{x}$-value, where the plastic wave meets the shock wave which freezes the rod, and the time $\underline{T}$ of impact. According to the theory, this quotient is equal to $c_1c_0/(c_0 - c_1)$, where c_1 is determined as before from a set of values v_1.

Figure 5. -- Here the abscissa is $\varepsilon-(v/c_0)$ and the ordinate is the velocity v_1 of the end of the rod.

It is seen that, except for Fig.5, the theoretical results are in much better agreement with the experimental results than Duwez's report indicated. We mention finally that the rupture velocity measured by Duwez was 171 ft/sec, whereas von Kármán's theoretical value was 180 ft/sec. Our results give the value 150 ft/sec.

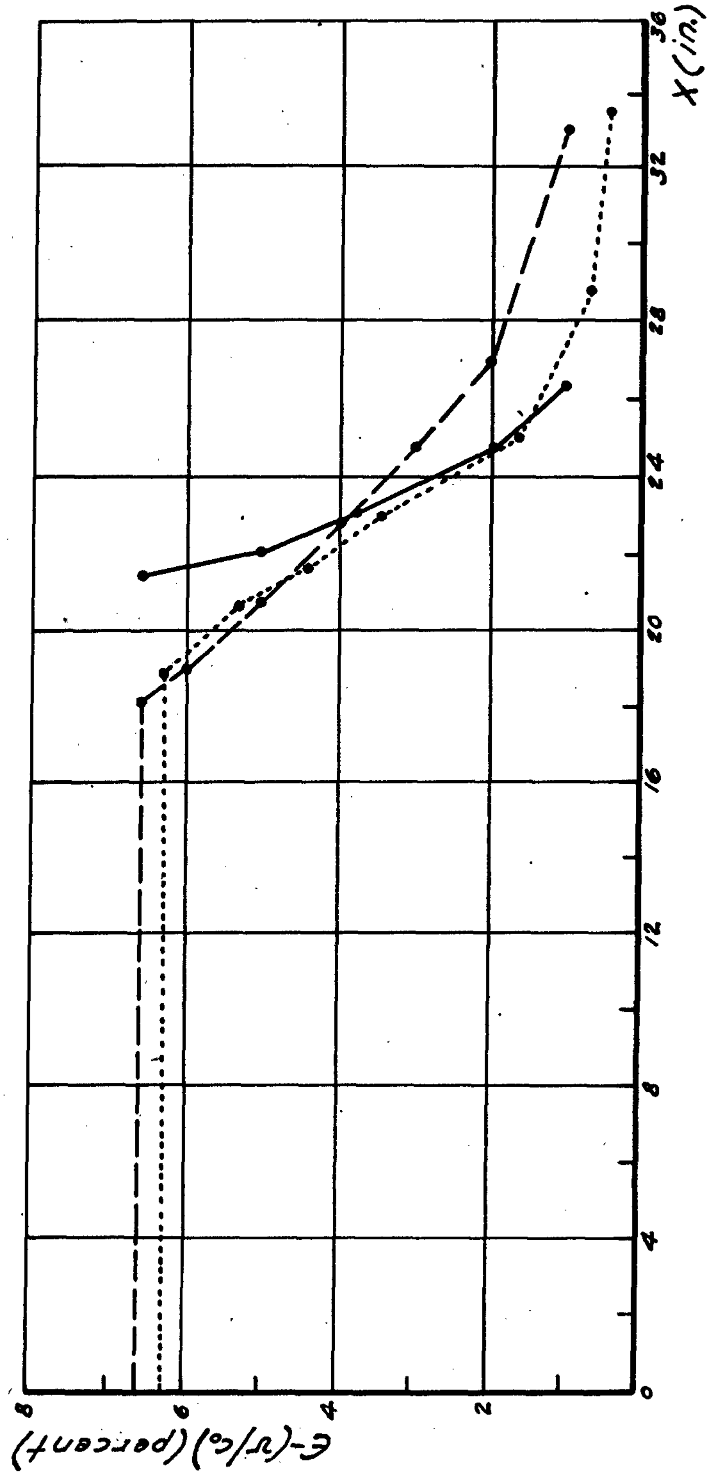

Fig. 12. $T = 1.63 \times 10^{-3}$ sec; $v = 92.50$ ft/sec. Solid curve, theoretical results [Duwez]; broken curve; experimental results [Duwez]; dotted curve, theoretical results.

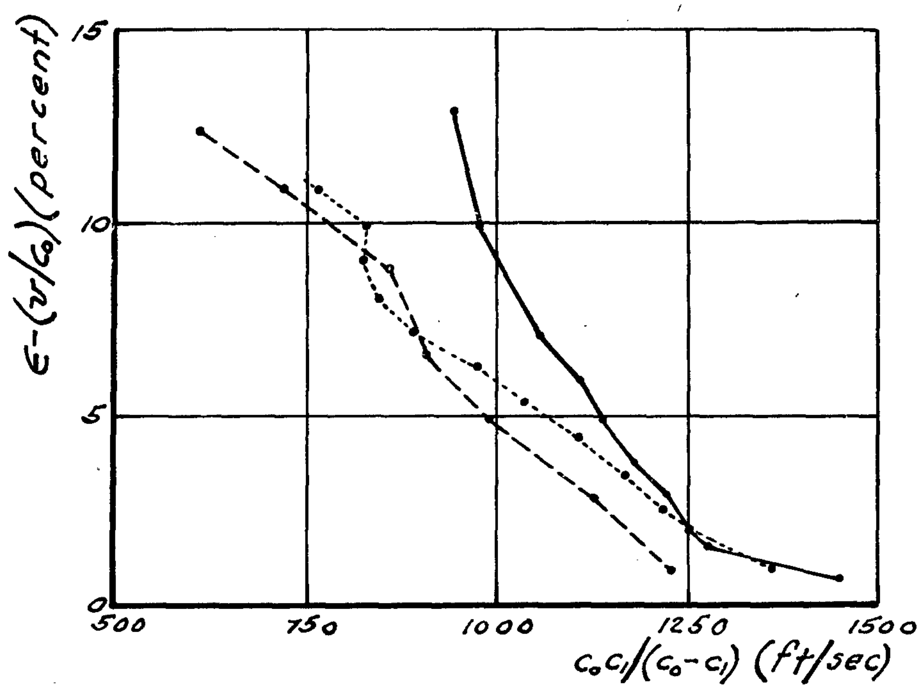

Fig. 13. Solid line, theoretical results [Duwez]; broken line, experimental results [Duwez]; dotted line, theoretical results.

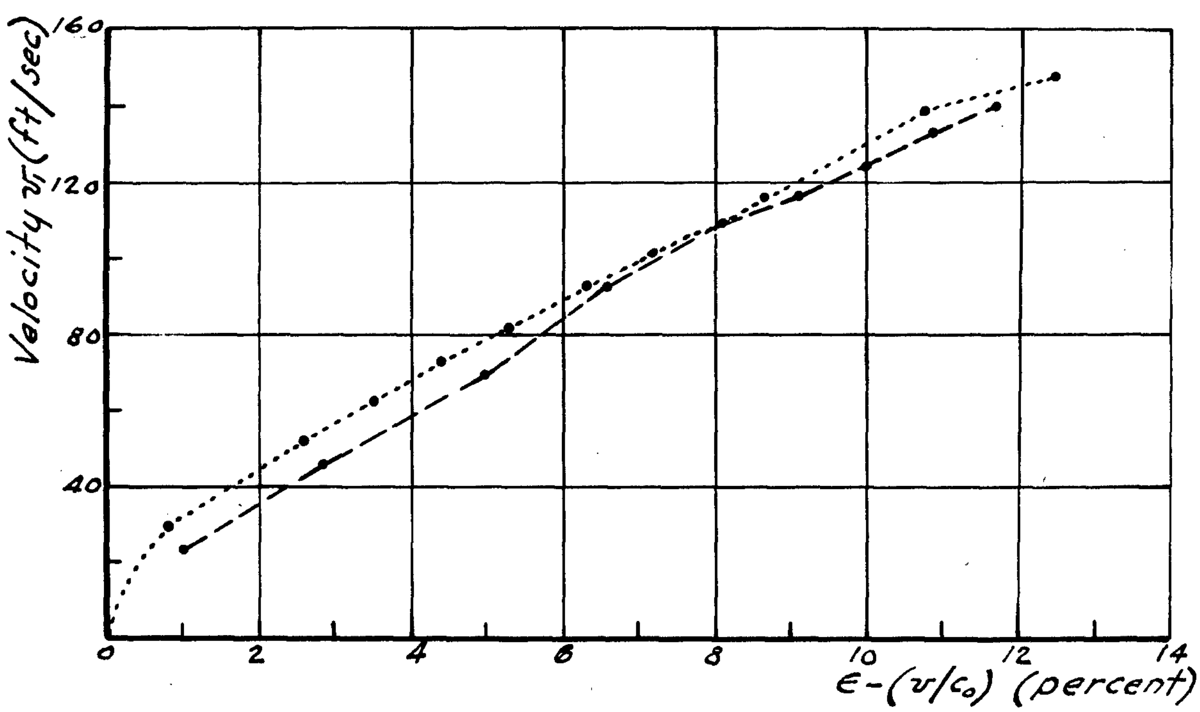

Fig. 5. Broken curve, experimental results [Duwez]; dotted curve, theoretical results.

OVER

RESTRICTED

ATI- 25159
REVISION (None)
ORIG. AGENCY NO. (None)
PUBLISHING AGENCY NO. OSRD-664

TITLE: A Note on Von Karman's Theory of the Propagation of Plastic Deformation in Solids

AUTHOR(S): Bohnenblust, H. F.

ORIGINATING AGENCY: Princeton University, Princeton, New Jersey

PUBLISHED BY: Office of Scientific Research and Development, NDRC, Div. A

DATE	DOC. CLASS.	COUNTRY	LANGUAGE	PAGES	ILLUSTRATIONS
June '42	Restr.	U.S.	Eng.	10	graphs

ABSTRACT:

Preliminary experiments were carried out by Pol. E. Duwez to check the formulas established by Th. von Karman in his theory of the propagation of plastic deformation in solids. Theorectical and experimental results are compared in Duwez's report, and it is pointed out that a discrepancy occurs. It is the purpose of the present memorandum to show that this discrepancy between theory and experiment is explained to a great extent if two changes are made in the interpretation of the theoretical results. These changes consist simply in avoiding two simplifications which were made by von Karman and Duwez, who underestimated their relative importance.

DISTRIBUTION: Copies of this report obtainable from Air Documents Division; Attn: MCIDXD

DIVISION: Ordnance and Armament ~~(22)~~ 22

SECTION: Armor ~~(5)~~ 2

ATI SHEET NO.: R-22-5-14

SUBJECT HEADINGS: Armor - Plastic deformation (11377); Metals - Plastic deformation (61072.5)

UNCLASSIFIED per Authority of OSRD List #29,
Dated 8-13 July 1946.